Report for 2004IN157B: Characterizing Errors in Distributed Hydrologic Modeling

There are no reported publications resulting from this project.

Report Follows

Title: Characterizing Errors in Distributed Hydrologic Modeling

Focus Category: Floods, Hydrology, Models, Non-Point Pollution, Surface Water

Keywords: Hydrologic Models, Runoff Estimation, Distributed Modeling, Error Estimation, Spatial Hydrology, Finite Element Analysis, GIS, Physically-Based Models, Pollution Prevention.

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Abstract:

Estimating surface water flow after a storm is critical in our humid region for flood analysis and water quality predictions among numerous other uses. Watershed hydrology is a mix of interacting processes that cuts across cascading and overlapping scales of time and space. Modeling these systems offers a unique tool for understanding and integrating the multi-scale processes when real observations and field monitoring fail due to the prohibitive time and cost required. To fully capture the real hydrologic system, modeling watershed hydrologic processes have survived years of empirical estimates. Although simple to develop and apply and are still in good use, empirical modeling fails to bring the accuracy, precision, and the needed details of the processes and their intermediate output. Physically-based and process oriented modeling has the potential to overcome this shortcoming; however, these more complex tools face numerous challenges. Addressing some of these challenges is the goal of this proposal. This research proposal is intended to establish the relationships among the hydrologic parameters and how they are impacted by the scales of the watershed and to evaluate the sources of errors associated with the numerical solution of overland flow. The result of this work will generate guidelines that will be implemented by hydrologic models to improve their efficiency and accuracy of simulation.

PROBLEM AND RESEARCH OBJECTIVES

The accurate determination of watershed runoff requires accurate determination of time-step for numerical simulations of kinematic wave shallow water equations in surface watershed hydrology. A time-step and Courant number based framework for error analysis for kinematic wave shallow water equations is developed for this project. The numerical simulation of watershed runoff hydrograph suffers from problems of estimability, non-uniqueness, multi-parameter heterogeneity and its representation, grid dependence and time-step. Therefore, two approaches namely- finite element method and finite volume method have been employed for the simulation of runoff hydrograph. Fourier (or von-Neumann) analysis is performed for Galerkin-formulated finite element system of error equations and Reynolds transport theorem based discrete form of difference equations using finite volume method.

The principal research objectives of the current project are to: (1) determine time-step for numerical simulations in two-dimensional watershed hydrology (2) perform stability analysis of shallow water equations using Fourier (or von-Neumann) analysis for consistent, lumped and upwind finite element schemes and finite volume schemes at node and at an element (3) compare the stability-based criteria for element and nodal based errors using finite element and finite volume methods (4) determine upper and lower bounds for time-step and Courant number for each finite element scheme using coefficient method (5) perform accurate analysis using eigen-value method (6) develop time-step and Courant number results for explicit, semi-implicit and implicit finite element and finite volume schemes.

METHODOLOGY AND PRINCIPAL FINDINGS

Fourier stability analysis is adapted for kinematic wave shallow water error equations. Fourier stability analysis requires approximation of nodal solution errors by grid discretization-dependent spatial and temporal error harmonics or individual Fourier components and estimation of magnitude of error amplification factor in wavenumber space. The Fourier analysis at the element level results in a single error equation for each element. Applying the Fourier stability analysis for all the element and nodal error equations, we obtain the amplification factor which shows evolution of error amplitude. The amplification factor is plotted for all the wavenumbers in wavenumber space, to check if the magnitude is lower than unit magnitude of amplitude to avoid numerical instability. If the amplification factor exceeds unit magnitude of amplitude for any wavenumber for any computational grid node, the time-step should be changed from its current value to ensure numerical stability. This is tantamount to changing the Courant number from its current value to other values for the same grid discretization. The same method is applied to all of the remaining element and nodal error equations using consistent, upwind, lumped for each of explicit, implicit, and semi-implicit finite element and finite volume schemes.

The principal research findings of the current project are summarized below:

1. Rigorous and conditional stability criteria are established for two-dimensional consistent, lumped and upwind formulations using explicit, semi-implicit and implicit finite element and finite volume schemes.
2. The error analysis at finite element level formulation is different from finite element nodal level formulation for two-dimensional problems. The

generalization of finite element nodal level error analysis is more accurate, as only nodal equations are truly solved, than finite element level error analysis. For unit magnitude of amplification factor, all of the finite element nodal evaluations yield the theoretical result $Tan\theta = i$ for all the methods consistent, upwind, and lumped using explicit, semi-implicit, and implicit schemes.

3. In the case of finite element method, the amplification factors determined using lower bound solutions of Courant numbers of coefficient method match closely with that of integer multiples of scalar eigen-value problem. The amplification factors determined using upper bound solutions of Courant numbers of coefficient method match closely with that of integer multiples of matrix eigen-value problem.
4. The nodal amplification factors expressed as a function of Courant number are evaluated to be equal for each consistent finite volume scheme- explicit, semi-implicit, and implicit. In the case of finite volume method, the amplification factors determined using coefficient method are compared with those obtained using exact eigen-value method. The close agreement between the amplification factors determined using integer multiples of Courant number of coefficient method and exact eigen-value method indicates that the solution given by coefficient method is a possible solution of kinematic system of shallow water equations.

SIGNIFICANCE OF PROJECT

The numerical simulation of kinematic wave shallow water equations requires large amount of two-dimensional watershed hydrological input data, surface roughness data, and storm data. For large-scale watershed runoff numerical simulations, we require accurate analysis of error equations using finite element or finite volume method. We would also need an accurate estimation of time-step to reduce simulation time and to obtain solution accuracy for kinematic wave shallow water equations. The significance of the project lies in establishing a theoretical framework for error analysis of systems of error equations using finite element and finite volume methods, in obtaining solutions for linearized shallow water equations in the form of amplification factors, in developing lower and upper bounds for scheme specific time-step, in upscaling of individual event or storm based time-step results from small to large-scale watershed runoff simulations, and in development of grid-discretization and time-step dependent simulation Courant numbers for computational, flood-plain watershed hydrology.

Three refereed publications and two conference presentations at the International ASAE meeting this July are in preparation to document these findings